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BRST CONSTRUCTION OF D-BRANES IN $SU(2)$ WZW MODEL.

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Abstract

BRST construction of D-branes in $SU(2)$ WZW model is proposed.

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1. Introduction

The role of D-branes [1] in the description of certain non perturbative degrees of freedom of strings is by now well established and the study of their dynamics has lead to many new insights into string and M-theory [2]. Much of this study was done in the large volume regime where geometric techniques provide reliable information. The extrapolation into the stringy regime usually requires boundary conformal field theory (CFT) methods. In this approach D-brane configurations are given by conformally invariant boundary states or boundary conditions. However the geometric properties of these configurations are well understood only for the case of flat and toric backgrounds where the CFT on the world sheet is a theory of free fields. Due to this reason many calculations concerning the scattering and emission of closed strings from D-branes can be given exactly (see for example [3], [4]). In particular, geometric features of the D-brane solutions of supergravity in flat 10-dimensional space-time were recovered from the boundary states in [5].

The class of rational CFT's gives the examples of curved string backgrounds where the construction of the boundary states leaving the whole chiral symmetry algebra or its subalgebra unbroken can be given in principle and the interaction of these states with closed strings can be calculated exactly. However the extraction of geometric features of boundary states is a problem as against the flat string background, because the boundary states are given in purely algebraic dates of the rational CFT of interest. One of the most important examples of this situation is given by Gepner models [6], where the boundary state approach developed in [7], [8], [9] has been used to get some of the geometric features of D-branes at small volume of the Calabi-Yau manifold.

WZW models on the compact groups is a subclass of rational CFT's where the chiral symmetry algebra is given by a Kac-Moody algebra and the space of states is given by the direct sum of integrable representations of the algebra [10]. D-branes in these models have been intensively studied in the last few years [11]- [18]. In particular, geometry of the boundary states which left unbroken Kac-Moody algebra of symmetries has been investigated in [14].

There is well known free field realization of WZW models [20]- [24] which allows to describe the irreducible representations of the Kac-Moody algebras, vertex operators and calculate the correlation functions. It is natural to ask whether we can extend the free field realization for boundary WZW models. To answer this question one needs to find first a free field construction of the boundary states.

This problem has been treated recently in [16]. In this work the known expression of a character of the irreducible representation of $su(2)$ Kac-Moody algebra via an alternating sum of the characters of Fock modules [20] has been used to represent Ishibashi state of the irreducible $\hat{su}(2)$ representation as a superposition of Isibashi states of the Fock modules. But this is certainly not the end of the construction because one needs to check that the boundary states constructed this way do not emit non-physical closed string states which are present originally in the free field realization. Similar to the bulk situation, the condition that non-physical states are not radiated by the boundary state is equivalent to BRST invariance of the state, with respect to the sum of BRST charges of Felder's resolution in the left- and right-moving sectors of the model. This condition which is of crucial importance in the free field construction of boundary states was not taken into account in [16].

In this note we study this problem for $SU(2)$ WZW model. In Section 2 we follow closely to the Section 3 of [16] and construct $\hat{su}(2)$ invariant Ishibashi states in the tensor product of Wakimoto modules from left- and right-moving sectors. Section 3 is the main part of the present note. In this section we construct Ishibashi state for each irreducible integrable $\hat{su}(2)$ representation using the superpositions of Ishibashi states of Wakimoto modules from Felder's resolution. We find the coefficients of the superposition imposing the BRST invariance condition which is similar to that of the bulk theory. Then we show that Ishibashi state constructed this way is not BRST exact and hence, represents a nontrivial homology class. At the end of Section 3 the boundary states in $SU(2)$ WZW model are constructed using the solution found by Cardy [25]. In Section 4 we consider briefly free field realization of the twisted boundary states and construct twisted BRST invariant boundary state for the case of Weyl reflection automorphism. Section 5 is devoted to discussions.

2. Ishibashi states in $\hat{sl}(2)$ Wakimoto modules

The standard $\hat{sl}(2)$-Wakimoto representation [26] in the left-moving sector is given by the set of free scalar fields $a(z)$, $a^+(z)$, $\phi(z)$:

$$
\begin{aligned}
a^+(z_1)a(z_2) &= -z_{12}^{-1} + reg., \\
\phi(z_1)\phi(z_2) &= -\ln z_{12} + reg.,
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
F &= -a^+, \\
H &= -2aa^+ + \imath\sqrt{2}\nu\partial\phi, \\
E &= a^2a^+ + k\partial a - \imath\sqrt{2}\nu a\partial\phi,
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
T &= T_\phi + T_a, \\
T_\phi = -\frac{1}{2}(\partial\phi)^2 + \imath\frac{1}{\sqrt{2}\nu}\partial^2\phi,\ & T_a = -a^+\partial a
\end{aligned} \tag{3}
$$

The parameter ν is related with the level k of the $\hat{sl}(2)$, $\nu^2 = k+2$. Similar to (1), (2), Wakimoto representation in the right-moving sector is given by the set of analogous formulas with the opposit sign in front of $\bar{\partial}\bar{\phi}$ [16]. The OPE's (1) give the commutation relations for the modes of the left-moving fields:

$$
\begin{aligned}
[a(n), a^+(m)] &= \delta_{n+m,0}, \\
[\phi(n), \phi(m)] &= -n\delta_{n+m,0}.
\end{aligned} \tag{4}
$$

(the similar relations are valid for the modes in the right-moving sector). Let us denote by $|P,l>$ the vacuum state of the fields a, a^+, ϕ which fulfills the relations

$$
\begin{aligned}
a(n)|P,l> &= 0,\ n > P, \\
a^+(n)|P,l> &= 0,\ n > -P-1,
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\phi(n)|P,l> &= 0,\ n > 0 \\
\phi(0)|P,l> &= \imath\frac{l}{\sqrt{2}\nu}|P,l>
\end{aligned} \tag{6}
$$

(P is a picture number [27]). This state has the following properties:

$$
\begin{aligned}
F(n)|P,l> &= 0,\ n > -1-P, \\
E(n)|P,l> &= 0,\ n > P, \\
H(n)|P,l> = 0,\ n > 0,\ H(0)|P,l> &= (2P-l)|P,l>, \\
L(n)|P,l> &= 0,\ n > 0, \\
L(0)|P,l> &= (-\frac{P(P-1)}{2} + \frac{l(l+2)}{4\nu^2})|P,l> .
\end{aligned} \tag{7}
$$

The state in the right-moving sector $|\bar{P},\bar{l}>$ fulfills the relations

$$\begin{aligned} \bar{a}(n)|\bar{P},\bar{l}>=0,\ n>\bar{P}, \\ \bar{a}^{+}(n)|\bar{P},\bar{l}>=0,\ n>-\bar{P}-1, \end{aligned} \tag{8}$$

$$\begin{aligned} \bar{\phi}(n)|\bar{P},\bar{l}>=0,\ n>0 \\ \bar{\phi}(0)|\bar{P},\bar{l}>=\imath\frac{\bar{l}}{\sqrt{2}\nu}|\bar{P},\bar{l}>. \end{aligned} \tag{9}$$

This state has the following properties:

$$\begin{aligned} \bar{F}(n)|\bar{P},\bar{l}>=0,\ n>-\bar{P}-1, \\ \bar{E}(n)|\bar{P},\bar{l}>=0,\ n>\bar{P}, \\ \bar{H}(n)|\bar{P},\bar{l}>=0,\ n>0,\ \bar{H}(0)|P,l>=(2\bar{P}+\bar{l})|\bar{P},\bar{l}>, \\ \bar{E}(n)|\bar{P},\bar{l}>=0,\ n>\bar{P}, \\ \bar{L}(n)|\bar{P},\bar{l}>=0,\ n>0,\ \bar{L}(0)|\bar{P},\bar{l}>=(-\frac{\bar{P}(\bar{P}+1)}{2}+\frac{\bar{l}(\bar{l}-2)}{4\nu^{2}})|\bar{P},\bar{l}>. \end{aligned} \tag{10}$$

The states with nonzero picture numbers can be described explicitly using the following representation of the fields a, a^{+} [27]. Let $\alpha(z),\beta(z)$ be the scalar free fields with the OPE's

$$\beta(z_1)\beta(z_2)=-\alpha(z_1)\alpha(z_2)=\ln z_{12}+r. \tag{11}$$

Then

$$a=\exp(\alpha-\beta),\ a^{+}=\exp(-\alpha)\partial\exp(\beta), \tag{12}$$

and the state $|P,l>$ corresponds to the vertex operator

$$A_P(z)V_l(z)=\exp(P\alpha(z))\exp(-\imath\frac{l}{\sqrt{2}\nu}\phi(z)). \tag{13}$$

A similar formula is valid for the state $|\bar{P},\bar{l}>$.

Let $W_{P,l}$ be the Fock module generated from $|P,l>$ by the creation operators. Analogously, we denote by $\bar{W}_{\bar{P},\bar{l}}$ the Fock module generated from $|\bar{P},\bar{l}>$ by the creation operators of the right-moving fields $\bar{a},\bar{a}^{+},\bar{\phi}$.

In the tensor product $W_{P,l}\otimes\bar{W}_{\bar{P},\bar{l}}$ we are going to construct Ishibashi state $|P,\bar{P},l,\bar{l}>>$ fulfilling on the boundary $z\bar{z}=1$ the relations (g-Ward identities)

$$\begin{aligned} (dzE(z)-d\bar{z}\bar{E}(\bar{z}))|P,\bar{P},l,\bar{l}>>=0, \\ (dzH(z)-d\bar{z}\bar{H}(\bar{z}))|P,\bar{P},l,\bar{l}>>=0, \\ (dzF(z)-d\bar{z}\bar{F}(\bar{z}))|P,\bar{P},l,\bar{l}>>=0. \end{aligned} \tag{14}$$

Thus, boundary CFT under consideration lives on the unit disc in z-plane.

It follows from the Sugawara formula that boundary conditions (14) are conformally invariant

$$((dz)^2T-(d\bar z)^2\bar T)|P,\bar P,l,\bar l>>=0, or$$
$$(L(n)-\bar L(-n))|P,\bar P,l,\bar l>>=0. \tag{15}$$

Let us solve (14) in terms of the free fields. Using (2), the similar formulas for the right-moving sector the third relation from (14) can be written as

$$(a^+(z)+\bar a^+(z))|P,\bar P,l,\bar l>>=0. \tag{16}$$

The second equation from (14) impose a relation between $a,\bar a,\phi,\bar\phi$. The equations which are consistent with (4), (16) are given by

$$\begin{aligned}(a(z)-\bar a(\bar z))|P,\bar P,l,\bar l>>=0,\\ (dz(\partial\phi-\imath\frac{\sqrt{2}}{\nu}(P+\bar P)z^{-1})+d\bar z\bar\partial\bar\phi)|P,\bar P,l,\bar l>>=0,\end{aligned} \tag{17}$$

The relations (17) can be rewritten in terms of the modes:

$$\begin{aligned}(a^+(n)+\bar a^+(-n))|P,\bar P,l,\bar l>>=0,\\ (a(n)-\bar a(-n))|P,\bar P,l,\bar l>>=0,\\ (\phi(n)-\bar\phi(-n)-\imath\frac{\sqrt{2}}{\nu}(P+\bar P)\delta_{n,0})|P,\bar P,l,\bar l>>=0.\end{aligned} \tag{18}$$

The solution of (18) is given by

$$\begin{aligned}\bar P=-1-P,\ \bar l=l+2,\\ |P,-1-P,l,l+2>>=\exp(\Sigma_{m=-P}a(-m)\bar a^+(-m)+\Sigma_{m=1+P}a^+(-m)\bar a(-m))\\ \exp(\Sigma_{m=1}\frac{1}{m}\phi(-m)\bar\phi(-m))|P,-1-P,l,l+2>.\end{aligned} \tag{19}$$

and it fulfills the equations (14).

One needs also to make a consistency check of the relations (18) and (15). Due to the commutation relations

$$\begin{aligned}[L(n),a^+(m)]=-ma^+(n+m),\\ [L(n),a(m)]=-(n+m)a(n+m),\\ [L(n),\phi(m)]=+\imath\frac{m(m-1)}{\sqrt{2}\nu}\delta_{n+m,0}-m\phi(n+m),\\ [\bar L(n),\bar\phi(m)]=-\imath\frac{m(m-1)}{\sqrt{2}\nu}\delta_{n+m,0}-m\bar\phi(n+m)\end{aligned} \tag{20}$$

we see that (15) is consistent with the relations (18) for the fields a,a^+. For the field ϕ we have

$$\begin{aligned}&[L(n)-\bar L(-n),\phi(m)-\bar\phi(-m)+\imath\sqrt{2}\nu\eta\delta_{m,0}]|P,\bar P,l,\bar l>>=\\ &-m(\phi(n+m)-\bar\phi(-n-m)+\imath\frac{\sqrt{2}}{\nu}\delta_{n+m,0})|P,\bar P,l,\bar l>>=0\end{aligned} \tag{21}$$

which is consistent with (18) and (19).

The boundary state (19) is a coherent state in the tensor product $W_{0,l} \otimes \bar{W}_{-1,l+2}$. In what follows we shall omit the right-moving indexes in the notation of Ishibashi states (19) because they are determined by the left-moving ones.

For each pair of Ishibashi states $|P, l >>$, $|P', l' >>$ one can calculate the cylinder partition function

$$Z_{(P',l'),(P,l)}(\tau, \theta) = << P', l'|q^{(L(0)-c/24)}u^{H(0)}|P, l >>, \qquad (22)$$

where $q = \exp(\imath 2\pi\tau)$, $u = \exp(\imath\pi\theta)$, τ and θ are the complex numbers ($Im\tau > 0$). After the calculation of the contractions we obtain

$$Z_{(P',l'),(P,l)} = \delta_{(P,l),(P',l')}$$
$$q^{\Delta(P,l)-c/24}u^{2P-l}\Pi_{m=-P}\frac{1}{(1-q^m u^2)}\Pi_{m=P+1}\frac{1}{(1-q^m u^{-2})}\Pi_{m=1}\frac{1}{(1-q^m)}, \qquad (23)$$

where $\Delta(P, l) = -\frac{P(P+1)}{2} + \frac{l(l+2)}{4\nu^2}$. It can be rewritten as the character $Ch(W_{0,l})$ of the Wakimoto module $W_{0,l}$

$$Z_{(P',l'),(P,l)} = (-1)^P\delta_{(P,l),(P',l')}$$
$$q^{\frac{l(l+2)}{4\nu^2}}u^{-l}\Pi_{m=0}\frac{1}{(1-q^m u^2)}\Pi_{m=1}\frac{1}{(1-q^m u^{-2})}\frac{1}{(1-q^m)} =$$
$$(-1)^P\delta_{(P,l),(P',l')}Ch(W_{0,l}). \qquad (24)$$

3. Ishibashi states in irreducible $\hat{su}(2)$-modules and boundary states in $SU(2)$ WZW model

In this section we represent the free field construction of Ishibashi and boundary states for irreducible integrable $\hat{su}(2)$ representations. Thus we restrict the level k and the momentum l to be nonnegative integer numbers such that $l < k + 1$.

To begin with the known BRST construction of the irreducible $\hat{su}(2)$ representations.

For each module $W_{0,l}$ one can associate the complex (Felder's resolution) [20], [21], [23]

$$\cdots \to C_l^{\frac{\infty}{2}-2} \to C_l^{\frac{\infty}{2}-1} \to C_l^{\frac{\infty}{2}} \to C_l^{\frac{\infty}{2}+1} \to C_l^{\frac{\infty}{2}+2} \to \cdots,$$
$$C_l^{\frac{\infty}{2}+2n} = W_{0,l_{2n}},\ l_{2n} = l - 2n\nu^2,$$
$$C_l^{\frac{\infty}{2}+2n+1} = W_{0,l_{2n+1}},\ l_{2n+1} = -l - 2 - 2n\nu^2,\ n \in Z, \qquad (25)$$

with the differentials (BRST operators)

$$d_{2n} = Q_{l+1},\ d_{2n+1} = Q_{\nu^2-l-1},$$
$$Q_m = \kappa_m \oint_\Gamma \prod_{i=1}^m dz_i S(z_i),\ \kappa_m = \frac{1}{m}\frac{\exp(\imath 2\pi\frac{m}{\nu^2}) - 1}{\exp(\imath 2\pi\frac{1}{\nu^2}) - 1}, \qquad (26)$$

where

$$S(z) = a^{+}(z) \exp(\imath \frac{\sqrt{2}}{\nu} \phi)(z) \tag{27}$$

and the integration contours Γ are chosen similar to [20]: the integrations contours over the variables $z_2, ..., z_m$ form a set nonintersecting curves going counterclockwise from z_1 to $\exp(\imath 2\pi) z_1$ and are localized in the neighborhood of the circle of radius $|z_1|$ centered at the origin, which is the z_1 integration contour.

The irreducible $\hat{su}(2)$-module M_l is given by the cohomology groups of the complex (25) [20], [21]

$$H^{\frac{\infty}{2}+i} = \delta_{i,0} M_l. \tag{28}$$

Thus the left-moving part of the Hilbert space of $SU(2)$ WZW model is given by BRST invariant states (modulo BRST exact). A similar statement takes place for the right-moving sector.

Due to this result one can write the character $Ch_l(q, u)$ of the module M_l as the Euler characteristic of the complex (25) [20]

$$Ch_l(q, u) = \Sigma_{k \in Z} (Ch(W_{0,l-2k\nu^2}) - Ch(W_{0,-l-2-2k\nu^2}))(q, u). \tag{29}$$

Hence, it is natural to suggest the free-field construction of the Ishibashi state $|M_l >>$ for the module M_l as a superposition of the states (19) with $P = 0$

$$|M_l >>= \Sigma_{k \in Z} (c_{2k} |0, l - 2k\nu^2 >> + c_{2k+1} |0, -l - 2 - 2k\nu^2 >>), \tag{30}$$

The coefficients c_n can be fixed partly from the condition

$$<< M_{l_1} |(-1)^f q^{(L(0)-c/24)} u^{H(0)} | M_{l_2} >>= \delta_{l_1,l_2} Ch_{l_1}(q, u), \tag{31}$$

where f is the "ghost" number operator associated with the complex (25) and determined by

$$f|v_n >= n|v_n >, \ |v_n > \in C_l^{\frac{\infty}{2}+n}. \tag{32}$$

Indeed, using (24) we obtain

$$\begin{gathered} << M_{l_1} |(-1)^f q^{(L(0)-c/24)} u^{H(0)} | M_{l_2} >>= \\ \Sigma_{k \in Z} (c_{2k}^2 Ch(W_{0,l_1-2k\nu^2}) - c_{2k+1}^2 Ch(W_{0,-l_1-2-2k\nu^2}))(q, u). \end{gathered} \tag{33}$$

Comparing with (29) we obtain

$$c_n = \pm 1, \ n \in Z. \tag{34}$$

Thus the state (30) is a good candidate for the free field realization of the Ishibashi state in $SU(2)$ WZW model [16]. It would be a genuine Ishibashi state for the module M_l if it did not radiate nonphysical closed string states which are present in the free field representation of the model. In other words, the overlap of this state with an arbitrary closed string state which does not belong to the Hilbert space of the WZW model should vanish. As we will see this condition can be formulated as a BRST invariance condition of the state (30) and it will fix the coefficients c_n up to the common sign.

To investigate BRST invariance of the state (30) one needs to consider the structure of the Wakimoto modules. We start with the module $W_{0,l}$. Its vacuum vector $|0,l>$ has the properties

$$\begin{aligned} F(0)|0,l> &= 0, \\ H(0)|0,l> &= -l|0,l>, \\ E(0)|0,l> &= la(0)|0,l>, \\ (E(0))^2|0,l> &= l(l-1)(a(0))^2|0,l>, \end{aligned} \tag{35}$$

where we have used the relation

$$[E(n),a(m)] = -(a)^2(n+m). \tag{36}$$

Thus we see that

$$(a(0))^{l+1}|0,l> \mathit{is\ a\ cosingular\ vector},\ (E(0))^{l+1}|0,l>=0. \tag{37}$$

We have also

$$\begin{aligned} (F(-1))^{\nu^2-l-1}|0,l> &= \\ (a^+(-1))^{\nu^2-l-1}|0,l> & \mathit{is\ a\ singular\ vector}. \end{aligned} \tag{38}$$

The whole structure of singular and cosingular vectors can be read of from the determinant formula for the module $W_{0,l}$ [19]

$$\begin{aligned} D_\mu &= \Pi_{m,n>0}[-l+m\nu^2-(n+1)]^{p(\mu-n(m\delta-\alpha))}, \\ C_\mu &= \Pi_{m,n>0}[l+(m-1)\nu^2-(n-1)]^{p(\mu-n((m-1)\delta+\alpha))}, \end{aligned} \tag{39}$$

where $\mu = r\delta+s\alpha, r=0,1,...,s\in Z$ is a positive root of $\hat{sl}(2)$, δ is the imaginary root, α is the positive root of $sl(2)$, $p(\mu)$ is the number of partitions of μ into the sum of positive roots of $\hat{sl}(2)$. The zeroes of C_μ correspond to the cosingular vectors, while the zeroes of D_μ correspond to the singular vectors. The complex (25) gives the structure of submodules of $W_{0,l}$.

Let us consider the structure of $\bar{W}_{-1,\bar{l}}$, $\bar{l}=l+2$. Its vacuum vector $|-1,\bar{l}>$ has the properties

$$\begin{aligned} \bar{E}(0)|-1,\bar{l}> &= 0, \\ \bar{H}(0)|-1,\bar{l}> &= (-2+\bar{l})|-1,\bar{l}>=l|-1,\bar{l}>, \\ \bar{E}(-1)|-1,\bar{l}> &= (k+2-\bar{l})\bar{a}(-1)|-1,\bar{l}>, \\ (\bar{E}(-1))^2|-1,\bar{l}> &= (k+2-\bar{l})(k+2-\bar{l}-1)|-1,\bar{l}>. \end{aligned} \tag{40}$$

So we find that

$$(\bar{a}(-1))^{\nu^2-\bar{l}+1}|-1,\bar{l}>=(\bar{a}(-1))^{\nu^2-l-1}|-1,\bar{l}> is\ a\ cosingular\ vector,$$
$$(\bar{E}(-1))^{\nu^2-l-1}|-1,\bar{l}>=0. \quad (41)$$

We have also

$$(\bar{F}(0))^{l+1}|-1,\bar{l}>=$$
$$(\bar{a}^{+}(0))^{l+1}|-1,\bar{l}> is\ a\ singular\ vector. \quad (42)$$

The determinant formula for the module $\bar{W}_{-1,\bar{l}}$ is given by [19]

$$C_\mu=\Pi_{m,n>0}[-l+m\nu^2-(n+1)]^{p(\mu-n(m\delta+\alpha))},$$
$$D_\mu=\Pi_{m,n>0}[l+(m-1)\nu^2-(n-1)]^{p(\mu-n((m-1)\delta-\alpha))}. \quad (43)$$

Hence, we see that $\bar{W}_{-1,l+2}$ is dual to $W_{0,l}$ [19] and its Felder's complex can be written as follows

$$\cdots\leftarrow\bar{C}_l^{\frac{\infty}{2}+2}\leftarrow\bar{C}_l^{\frac{\infty}{2}+1}\leftarrow\bar{C}_l^{\frac{\infty}{2}}\leftarrow\bar{C}_l^{\frac{\infty}{2}-1}\leftarrow\bar{C}_l^{\frac{\infty}{2}-2}\leftarrow\cdots,$$
$$\bar{C}_l^{\frac{\infty}{2}+2n}=\bar{W}_{-1,\bar{l}_{2n}},\ \bar{l}_{2n}=2+l+2n\nu^2,$$
$$\bar{C}_l^{\frac{\infty}{2}+2n-1}=\bar{W}_{-1,\bar{l}_{2n-1}},\ \bar{l}_{2n-1}=-l+2n\nu^2,\ n\in Z, \quad (44)$$

where the differentials are given by

$$\bar{d}_{2n-1}=\bar{Q}_{l+1},\ \bar{d}_{2n}=\bar{Q}_{\nu^2-l-1},$$
$$\bar{Q}_m=\kappa_m\oint_{\Gamma^{-1}}\prod_{i=1}^{m}d\bar{z}_i\bar{S}(\bar{z}_i), \quad (45)$$

where $n\in Z$,

$$\bar{S}(\bar{z})=\bar{a}^{+}(\bar{z})\exp(-\imath\frac{\sqrt{2}}{\nu}\bar{\phi})(\bar{z}), \quad (46)$$

and the integration contours Γ^{-1} have opposite orientation and opposite ordering with respect to Γ.

Next we form a tensor product of the complexes (25) and (44):

$$\cdots\rightarrow A_l^{\frac{\infty}{2}-2}\rightarrow A_l^{\frac{\infty}{2}-1}\rightarrow A_l^{\frac{\infty}{2}}\rightarrow A_l^{\frac{\infty}{2}+1}\rightarrow A_l^{\frac{\infty}{2}+2}\rightarrow\cdots,$$
$$A_l^{\frac{\infty}{2}+p}=\bigoplus_{n+m=p}C_l^{\frac{\infty}{2}+n}\otimes\bar{C}_l^{\frac{\infty}{2}+m} \quad (47)$$

with the differential D defined by

$$D_p(v_n\otimes\bar{v}_m)=d_nv_n\otimes\bar{v}_m+(-1)^nv_n\otimes\bar{d}_m\bar{v}_m,\ n+m=p \quad (48)$$

where $v_n\otimes\bar{v}_m$ is an arbitrary element from $A_l^{\frac{\infty}{2}+p}$.

The cohomology groups of the complex (47) are given by

$$\mathbf{H}^{\frac{\infty}{2}+i} = \delta_{i,0} M_l \otimes M_l^*, \tag{49}$$

where M_l^* is dual module to M_l.

The Ishibashi state we are looking for can be considered as a linear functional on the Hilbert space of $SU(2)$ WZW model, then it has to be an element from homology group $\mathbf{H}_{\frac{\infty}{2}}$. Therefore, the BRST invariance condition for the state can be formulated as follows.

Let us define the action of the differential D on the state $|M_l >>$ by the formula

$$<< D^* M_l | v_n \otimes \bar{v}_{p-n} > \equiv << M_l | D_p | v_n \otimes \bar{v}_{p-n} >, \tag{50}$$

where $v_n \otimes \bar{v}_{p-n}$ is an arbitrary element from $A_l^{\frac{\infty}{2}+p}$. Then, BRST invariance condition means that

$$D^* | M_l >> = 0. \tag{51}$$

Proposition.

The superposition (30) satisfy the BRST invariance condition (51) if the coefficients c_n obey the following equation

$$(-1)^{-n} c_{-n} + c_{1-n} = 0. \tag{52}$$

Note that the last relation is consistent with (34) and we obtain two sets of solutions of (52)

$$\begin{gathered} c_{2m} = (-1)^m, \ c_{2m+1} = -(-1)^m, \ or \\ c_{2m} = -(-1)^m, \ c_{2m+1} = (-1)^m, \ m \in Z. \end{gathered} \tag{53}$$

Now we move on to the prove of the Proposition.

Let us consider an arbitrary state

$$|w_{-n} \otimes \bar{w}_{-m} > \in A_l^{\frac{\infty}{2}-n-m} \tag{54}$$

from the complex (47). In view of (30) only the states from $A_l^{\frac{\infty}{2}}$ have nonzero overlap with Ishibashi state $|M_l >>$. Because of the differential D rises the ghost number by one we have to put in (54) $-n-m=-1$. So one needs to show that

$$\begin{gathered} << M_l | D_{-1} | w_{-n} \otimes \bar{w}_{-1+n} > \equiv \\ << M_l | d_{-n} + (-1)^{-n} \bar{d}_{-1+n} | w_{-n} \otimes \bar{w}_{-1+n} > = 0. \end{gathered} \tag{55}$$

We will imply in the following that the state (54) corresponds to the field $(D_{-1}(w_{-n} \otimes \bar{w}_{-1+n}))(z,\bar{z})$ which is placed at the center $z=\bar{z}=0$ of the disk.

Let us consider the first term of (55).

$$<< M_l|d_{-n}|w_{-n} \otimes \bar{w}_{-1+n} >= \kappa_{-n} << M_l| \oint_\Gamma \prod_{i=1}^N dz_i S(z_i)|w_{-n} \otimes \bar{w}_{-1+n} >=$$

$$\kappa_{-n} << M_l| \oint_{\gamma_1} dz_1 S(z_1) \int_{\Gamma'} \prod_{i=2}^N dz_i S(z_i)|w_{-n} \otimes \bar{w}_{-1+n} > . \quad (56)$$

In this formula N is determined by (26). Recall also that we can chose z_1 integration contour γ_1 as the unit circle centered at $z = \bar{z} = 0$ (so it coincide with the boundary of the disk) such that the integration contours Γ' form a set of non-intersecting nested curves going counterclockwise from z_1 to $\exp(\imath 2\pi)z_1$ and are localized in the neighborhood of the contour γ_1. When acting on $|w_{-n} \otimes \bar{w}_{-1+n} >$ the BRST current

$$J(z_1) = S(z_1) \int_{\Gamma'} \prod_{i=2}^N dz_i S(z_i) \quad (57)$$

is single valued around the center of the disk.

To prove (55) we change first $dz_1 S(z_1)$ taking into account normal ordering in the $\exp(\imath\frac{\sqrt{2}}{\nu}\phi)$ and using the relations (18)

$$<< M_l|dz_1 S(z_1) =$$

$$<< M_l|dz_1 a^+(z_1) \exp(\imath\frac{\sqrt{2}}{\nu}\phi_<(z_1)) \exp(\imath\frac{\sqrt{2}}{\nu}\phi_0) z_1^{\imath\frac{\sqrt{2}}{\nu}\phi(0)} \exp(\imath\frac{\sqrt{2}}{\nu}\phi_>(z_1)) =$$

$$<< M_l| \exp(\imath\frac{\sqrt{2}}{\nu}(\phi_0 + \bar{\phi}_0)) d\bar{z}_1 \bar{S}(\bar{z}_1). \quad (58)$$

Then, deform the first integration contour from Γ' towards the boundary $z\bar{z} = 1$ and apply the relations (18). Note that we have no relations for the modes ϕ_0, $\bar{\phi}_0$ canonically conjugated to the momentum modes $\phi(0)$, $\bar{\phi}(0)$. So we keep the ϕ_0 unchanged during this process. Thus we obtain

$$<< M_l|dz_1 J(z_1)|w_{-n} \otimes \bar{w}_{-1+n} >=$$

$$\kappa_{-n} << M_l| \exp(\imath\frac{\sqrt{2}}{\nu}2(\phi_0 + \bar{\phi}_0)) d\bar{z}_1 \bar{S}(\bar{z}_1) \oint_{\gamma_2} d\bar{z}_2 \bar{S}(\bar{z}_2) \int_{\Gamma''} \prod_{i=3}^N dz_i S(z_i)$$

$$|w_{-n} \otimes \bar{w}_{-1+m} > . \quad (59)$$

Now we can deform the integration contour γ_2 towards the origin such that $|\bar{z}_2| < |z_N|$.

Repeating the same procedure for other contours we obtain

$$<< M_l|d_{-n}|w_{-n} \otimes \bar{w}_{-1+n} >=$$

$$\kappa_{-n} << M_l| \exp(\imath\frac{\sqrt{2}}{\nu}N(\phi_0 + \bar{\phi}_0)) \oint_{\Gamma^{-1}} \prod_{i=N}^1 d\bar{z}_i \bar{S}(\bar{z}_i)|w_{-n} \otimes \bar{w}_{-1+m} >=$$

$$c_{1-n} << \bar{l}_{-1+n}, l_{1-n}, -1, 0| \exp(\imath\frac{\sqrt{2}}{\nu}N(\phi_0 + \bar{\phi}_0))|w_{-n} \otimes \bar{d}_{-1+n}\bar{w}_{-1+n} >, \quad (60)$$

where Γ^{-1} denotes the set of nested contours with opposite orientation and opposite ordering.

The operator $\exp(\imath\frac{\sqrt{2}}{\nu}N(\phi_0+\bar{\phi}_0))$ only shifts the ghost numbers of the state $|w_{-n}\otimes\bar{d}_{-1+n}\bar{w}_{-1+n}>$, such that the only nonzero pairing appears for $<<\bar{l}_{-1+n},l_{1-n},-1,0|$. From the other hand

$$\begin{aligned} &<<M_l|\bar{d}_{-1+n}|w_{-n}\otimes\bar{w}_{-1+n}>= \\ &c_{-n}<<\bar{l}_n,l_{-n},-1,0|w_{-n}\otimes\bar{d}_{-1+n}\bar{w}_{-1+n}>. \end{aligned}\tag{61}$$

Comparing these formulas we conclude that (55) will be satisfied iff (52) is fulfilled. It proves the Proposition.

To complete the free field construction of the Ishibashi state one needs to check that $|M_l>>$ is not BRST exact state. It can be done by projecting the state $|M_l>>$ onto an element from the cohomology class $\mathbf{H}^{\frac{\infty}{2}}$. Let us consider the case when $l>0$ and choose the representative from the zero-level states of $M_{l'}\otimes M^*_{l'}$. It can be written as $(E(0))^n(\bar{F}(0))^m|0,-1,l',l'+2>$. Then

$$\begin{aligned} &<<M_l|(E(0))^n(\bar{F}(0))^m|0,-1,l',l'+2>= \\ &(-1)^n l(l-1)...(l-n+1)\delta_{n,m}\delta_{l,l'}c_0. \end{aligned}\tag{62}$$

For the case $l=0$ we choose $(F(-1))^n(\bar{E}(-1))^m|0,-1,l',l'+2>, n<k-l'+1$ as a representative from $M_{l'}\otimes M^*_{l'}$. Projecting $|M_0>>$ onto the representative we find the expression like (62). Therefore, we see that result is not zero and hence the Ishibashi state $|M_l>>$ defines a homology class. Moreover, similar to [14] the projection is given by the calculation of diagonal matrix elements in the irreducible $su(2)$- representation if we put

$$c_0=1. \tag{63}$$

In view of the Proposition we can obtain free field representation of the boundary states in $SU(2)$-WZW model just applying the formula found by Cardy [25]:

$$|B_l>>=\sum_{j\in I}\frac{S_{lj}}{\sqrt{S_{0j}}}|M_j>>, \tag{64}$$

where $S_{lj}, j,l\in I$ is the matrix of modular transformation of $\hat{su}(2)$-characters:

$$S_{lj}=\frac{\sqrt{2}}{\nu}\sin(\pi(j+1)(l+1)/\nu^2). \tag{65}$$

Using (64), (62) and (63) it is easy to recover the wave function of the $SU(2)$ boundary state found in [14].

4. Free field realization of twisted boundary states.

In this section we consider briefly the twisted boundary conditions of automorphism type. The theory treating such boundary conditions for arbitrary CFT's has been developed in [28] and some applications of these results to WZW models has been considered in [29]. The automorphisms which are interested to us here are induced by automorphisms of the algebra $su(2)$. They are all inner automorphisms and can be represented by the adjoint action of the group $SU(2)$. We consider here only the case when it is given by the Weyl reflection r of $su(2)$. The generalization is straightforward.

Hence, we have to construct first the twisted Ishibashi states $|0,-1,l,\bar{l}>>^r$ fulfilling on the boundary $z\bar{z}=1$ the relations

$$\begin{aligned}(dz(r.E)(z) - d\bar{z}\bar{E}(\bar{z}))|0,-1,l,\bar{l}>>^r &= 0,\\ (dz(r.H)(z) - d\bar{z}\bar{H}(\bar{z}))|0,-1,l,\bar{l}>>^r &= 0,\\ (dz(r.F)(z) - d\bar{z}\bar{F}(\bar{z}))|0,-1,l,\bar{l}>>^r &= 0,\end{aligned} \tag{66}$$

where

$$(r.E)(z) = -F(z),\ (r.H)(z) = -H(z),\ (r.F)(z) = -E(z). \tag{67}$$

The relations (66) are equivalent to

$$\begin{aligned}(\bar{a}^+(z) + a^2a^+(z) + k\partial a(z) - \imath\sqrt{2}\nu a\partial\phi(z))|0,-1,l,\bar{l}>>^r &= 0,\\ (\bar{a}(z) + a^{-1}(z))|0,-1,l,\bar{l}>>^r &= 0,\\ (\partial\bar{\phi}(z) - \partial\phi(z) - \imath\sqrt{2}\nu(a^{-1}\partial a(z) + \frac{1}{\nu^2}z^{-1}))|0,-1,l,\bar{l}>>^r &= 0,\end{aligned} \tag{68}$$

where $a^{-1}(z) \equiv \exp(-\alpha+\beta)(z)$. In terms of the modes these equations are given by

$$\begin{aligned}(\bar{a}^+(n) - E(-n))|0,-1,l,\bar{l}>>^r &= 0,\\ (\bar{a}(n) + a^{-1}(-n))|0,-1,l,\bar{l}>>^r &= 0,\\ (\bar{\phi}(n) - \phi(-n) - \imath\sqrt{2}\nu((a^{-1}\partial a)(-n) + \delta_{-n,0}\frac{1}{\nu^2}))|0,-1,l,\bar{l}>>^r &= 0.\end{aligned} \tag{69}$$

One can easily check that

$$\begin{aligned}\exp(E(0))\exp(-F(0))\exp(E(0))a(z) &= -a^{-1}(z),\\ \exp(E(0))\exp(-F(0))\exp(E(0))a^+(z) &= -F(z),\\ \exp(E(0))\exp(-F(0))\exp(E(0))\partial\phi(z) &= \partial\phi(z) + \imath\sqrt{2}\nu a^{-1}\partial a(z).\end{aligned} \tag{70}$$

Thus the Ishibashi state we are going to construct has to act on the fields $\bar{a}, \bar{a}^+, \bar{\phi}$ as the composition

$$(\bar{a}, \bar{a}^+, \bar{\phi}) \rightarrow (a, a^+, \phi) \rightarrow r(a, a^+, \phi), \tag{71}$$

where

$$r = \exp(E(0))\exp(-F(0))\exp(E(0)). \tag{72}$$

Then, in according to (71) we can write

$$|0,-1,l,\bar{l}>>^r = (r\otimes 1)|0,l>> . \tag{73}$$

It is obvious that we can use instead of r defined by (72) the operator $\bar{r}$ which acts in the right-moving sector and defined by

$$\bar{r} = \exp(\bar{F}(0)) \exp(-\bar{E}(0)) \exp(\bar{F}(0)). \tag{74}$$

Then one can rewrite (73) in the equivalent form

$$|0, -1, l, \bar{l} >>^r = (1 \otimes \bar{r})|0, l >> . \tag{75}$$

It is easy to see also that operators r and $\bar{r}$ change the pictures, the picture of the state $r \otimes 1|0, l >>$ is $(-1, -1)$ and the picture of the state $1 \otimes \bar{r}|0, l >>$ is $(0, 0)$. It enables us to conclude that

$$\begin{aligned} (r \otimes 1)|0, l >> &\in W_{-1,-2-l} \otimes \bar{W}_{-1,l+2}, \\ (1 \otimes \bar{r})|0, l >> &\in W_{0,l} \otimes \bar{W}_{0,-l}. \end{aligned} \tag{76}$$

The next step is to construct twisted Ishibshi states for the irreducible $\hat{su}(2)$ modules. Because of the differentials of the Felder's complexes are invariant with respect to the left-moving and right-moving $\hat{su}(2)$ algebras one can easily extend the construction of the Section 3 to the case of twisted boundary conditions.

Let us consider for example the representation (75) for the twisted Ishibashi state in Wakimoto modules. In this case the structures of modules $W_{0,l}$ and $\bar{W}_{0,-l}$ coincide so that complex in the right-moving sector coincides with the complex (25) in the left-moving sector. Similar to (47) we can form the double complex which calculates the tensor product of irreducible representations. Thus it is easy to see that state

$$(1 \otimes \bar{r})|M_l >>, \tag{77}$$

where $|M_l >>$ is given by (30), (52), is BRST invariant and gives free field realization of twisted Ishibashi state in $SU(2)$ WZW model. It is obvious also that the state

$$(1 \otimes \bar{r})|B_l >>, \tag{78}$$

where $|B_l >>$ is given by (64) represents twisted boundary state of the model.

5. Discussion

In this note we have constructed $\hat{su}(2)$ and BRST invariant Ishibashi states in $SU(2)$ WZW models using free field realization of the bulk WZW model. Each Ishibashi state of the model is given by infinite superposition of Ishibashi states of Wakimoto modules, forming Felder's complex for irreducible $\hat{su}(2)$ module. It is shown that coefficients of the superposition are fixed uniquely by the BRST invariance condition and the Ishibashi state constructed this way is not BRST exact and hence represents nontrivial homolgy class. We group these free field realized Ishibashi states into the boundary states of $SU(2)$ WZW model using the solution found by Cardy.

Due to BRST invariance the boundary states do not radiate non-physical closed string states (which are present originally in the free field realized WZW model) and its wave functions coincide with the wave functions of conjugacy classes found in [14].

Using free field representation of the automorphism group of $su(2)$ algebra we have constructed also the twisted automorphism type boundary states when the automorphism is given by Weyl reflection.

Note also that free field representation for the character of the irreducible $\hat{su}(2)$ module (33)

very close to the open-string Witten index [30], [8] which has been widely discussed in the literature in context of D-branes on Calabi-Yau manifolds. Indeed, using (33, 64) and Verlinde formula one can obtain:

$$\begin{aligned} &<< B_{l_1}|(-1)^f q^{(L(0)-c/24)} u^{H(0)}|B_{l_2} >>= \\ &\sum_{j\in I} \frac{S_{l_1 j}}{S_{0j}} \frac{S_{l_2 j}}{S_{0j}} S_{0j} Ch_j(q,u) = \sum_{j,l\in I} N^l_{l_1 l_2} S_{lj} Ch_j(q,u) = \\ &\sum_{l\in I} N^l_{l_1 l_2} Ch_l(Sq, Su). \end{aligned} \tag{79}$$

Thus in the open string sector we have

$$Tr_{\Omega_{l_1 l_2}}(-1)^{f^{op}} \tilde{q}^{(L^{op}(0)-c/24)} \tilde{u}^{H^{op}(0)} = \sum_{l\in I} N^l_{l_1 l_2} Ch_l(\tilde{q},\tilde{u}), \tag{80}$$

where $\Omega_{l_1 l_2}$ is Hilbert space in the open string sector, f^{op}, $L^{op}(0)$, $H^{op}(0)$, are the corresponding operators, and $\tilde{q} = \exp(-\imath\frac{2\pi}{\tau})$, $\tilde{u} = \exp(\imath\frac{\pi\theta}{\tau})$.

The right hand side of this equality is given by the character-valued index of the BRST operator in the space Ω_{l_1,l_2}. This expression is similar to that obtained for example in [8], [31], [32] for N=2 superconformal field theories. It would be interesting to find geometric interpretation of (80).

We close with a brief discussion of some directions to develop. The first one is a generalization of BRST construction of Ishibashi states for higher rank groups. It would also be interesting to join BRST construction of Ishibashi states in WZW models and quantum Drinfeld-Sokolov reduction [34] to give free field realization of Ishibashi and boundary states in the CFT's with W-algebra of symmetries [35]. The next obviously important direction is an extension of the construction to the case of supersymmetric WZW models and N=2 coset models.

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